

Quantum field theory for the blind: Feynman diagrams in the Triangle language

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Abstract

Feynman diagrams are inherently visual representations of particle interactions in quantum field theory. To make these diagrams accessible to users of WinTriangle, a scientific word processor for the blind, a representation of Feynman diagrams is introduced in the Triangle language.

1 The Rules

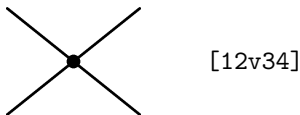
Feynman diagrams present a unique challenge in any effort to convey processes in quantum field theory to a blind student or researcher. In this paper, we present a Triangle-based syntax for describing these diagrams. The syntax for a Feynman diagram is as follows.

- Begin the diagram with a [
- List the first vertex as \langle lines going into the vertex $\rangle v \langle$ lines coming out of the vertex \rangle
- Separate the vertices with a |
- \vdots
- Separate the vertices with a |
- List the last vertex as \langle lines going into the vertex $\rangle v \langle$ lines coming out of the vertex \rangle
- End the diagram with a]

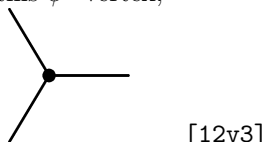
Every propagator (line) of each type must have a unique identifier; the identifiers may be either numbers $(1, 2, \dots)$ or momenta (p, p', k, k', \dots) . Therefore, these identifiers never appear more than twice in a diagram. The identifier will appear once if the line corresponds to an incoming or outgoing particle, and it will appear exactly twice if it is an internal line (emission from a vertex and absorption into a vertex).

2 Examples

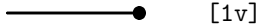
When all particles are of the same type, a simple number may be used to identify each particle as in this ϕ^4 interaction vertex,



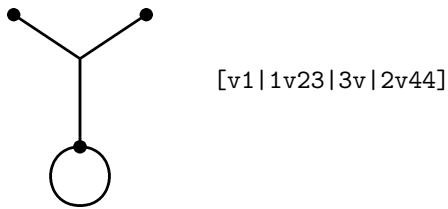
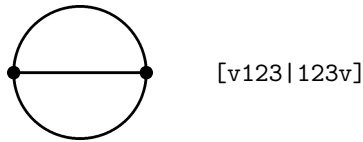
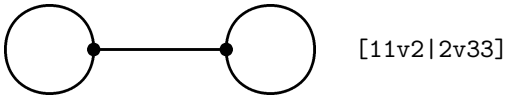
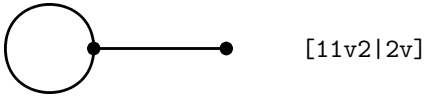
this ϕ^3 vertex,



or this simple tadpole.



Some simple vacuum diagrams are given below as further examples.



In the preceding diagrams, the time direction was irrelevant. Most field theorists use either a left-to-right or bottom-to-top time direction. In all subsequent diagrams, we will use a bottom-to-top time direction. That is, time increases from the bottom of the diagram to the top (incoming particles on the bottom, outgoing particles on the top).

In the examples so far, there were only bosons of the same type. In diagrams with fermions, further notation is required. Each fermionic line will be described with an `f`, for fermion, followed by its identifier (number or momentum) and other defining characteristics inside parentheses. For example, instead of using `1` to identify the lower-left line in the diagram below, we use, in LaTeX, `f(p,\arrowin,\incoming)`. This tells us that this line is a fermion, has momentum p , its arrow is pointing *into the vertex*, and that it is an *incoming particle*.

The LaTeX commands `\arrowin`, `\arrowout`, `\incoming`, and `\outgoing` will compile correctly in the LaTeX2Tri converter to produce corresponding RTF symbols. On the other hand, these commands will confuse the regular latex compiler. If a sighted person wishes to check their work in PDF or Postscript before converting to Triangle, then the following code may be added to the top of the LaTeX file:

```

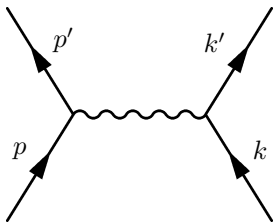
\def\arrowin{\text{arrowin}}
\def\arrowout{\text{arrowout}}
\def\incoming{\text{ingoing}}
\def\outgoing{\text{outgoing}}

```

Before converting to Triangle with LaTeX2Tri, these lines must be commented out.

If the fermion is an electron or positron, then e^- or e^+ may be used instead of f , respectively. A boson (most likely a photon) will be denoted by the letter g or the Greek letter γ , the standard symbol for a photon. (The letter g was chosen over the letters p (for “photon”) or b (for “boson”) because the latter letters sound similar to v when spoken by the computer.)

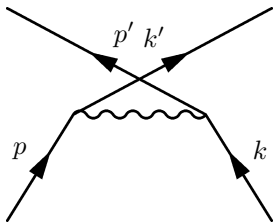
The following two diagrams contribute at lowest order to electron-electron scattering. Note that no identifier has been given for the photon because it is the only g in the diagram, so there will be no confusion.



```

[f(p,\arrowin,\incoming)vf(p',\arrowout,\outgoing)g|
gf(k,\arrowin,\incoming)vf(k',\arrowout,\outgoing)]

```

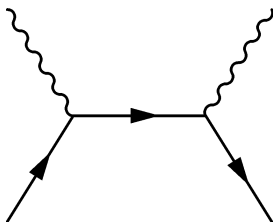


```

[f(p,\arrowin,\incoming)vf(k',\arrowout,\outgoing)g|
gf(k,\arrowin,\incoming)vf(p',\arrowout,\outgoing)]

```

The following diagram represents electron-positron annihilation. Note that \arrowin and \arrowout specify which direction the arrow is *relative to the vertex*, not relative to the time direction (i.e., incoming or outgoing particle)! It is very important not to confuse the two concepts.

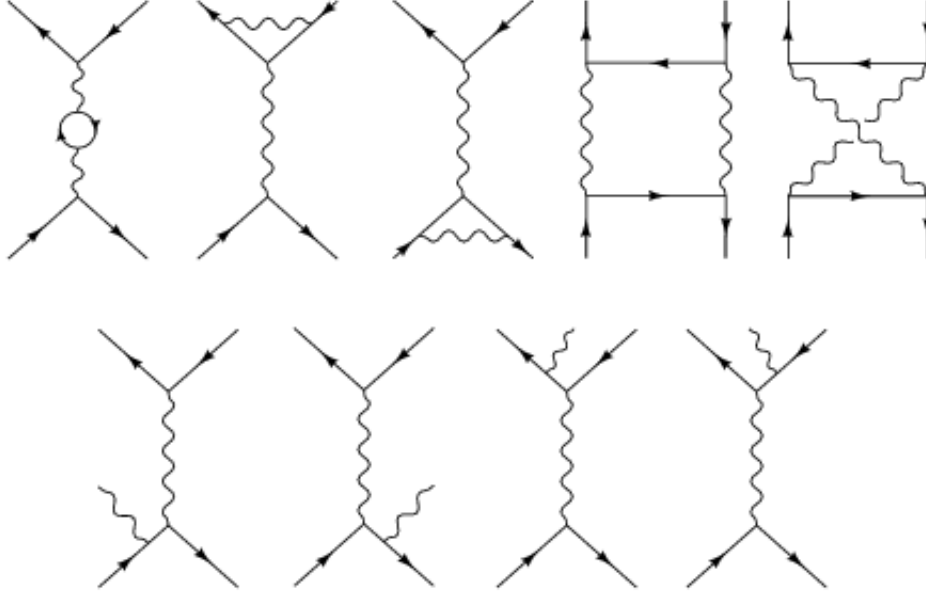


```

[f(1,\arrowin,\incoming)vg(1,\outgoing)f(2,\arrowout)|
f(2,\arrowin)vf(3,\arrowout,\incoming)g(2,\outgoing)]

```

Given the following grouping of Feynman diagrams that contribute to the α^3 term in the $e^+e^- \rightarrow \mu^+\mu^-$ cross section, the corresponding Triangle output is shown. (An error in Acrobat Reader causes the second half of this page to not display on the screen, but it will appear when printed.)



$$[e^-(\bar{f}, \bar{m})e^+(\underline{f}, \bar{m})\nu g(1)|g(1)\nu f(1, \underline{f})f(2, \bar{f})|f(1, \bar{f})f(2, \underline{f})\nu g(2)|g(2)\nu\mu^-(\underline{f}, \underline{m})\mu^+(\bar{f}, \underline{m})]$$

$$[e^-(\bar{f}, \bar{m})e^+(\underline{f}, \bar{m})\nu g(1)|g(1)\nu f(1, \underline{f})f(2, \bar{f})|f(1, \bar{f})\nu g(2)\mu^-(\underline{f}, \underline{m})|f(2, \underline{f})g(2)\nu\mu^+(\bar{f}, \underline{m})]$$

$$[e^-(\bar{f}, \bar{m})\nu g(1)f(1, \underline{f})|g(1)e^+(\underline{f}, \bar{m})\nu f(2, \bar{f})|f(1, \bar{f})f(2, \underline{f})\nu g(2)|g(2)\nu\mu^-(\underline{f}, \underline{m})\mu^+(\bar{f}, \underline{m})]$$

$$[e^-(\bar{f}, \bar{m})\nu g(1)f(1, \underline{f})|f(1, \bar{f})\nu e^+(\underline{f}, \bar{m})g(2)|g(2)\nu f(2, \underline{f})\mu^+(\bar{f}, \underline{m})|g(1)f(2, \bar{f})\nu\mu^-(\underline{f}, \underline{m})]$$

$$[e^-(\bar{f}, \bar{m})\nu g(1)f(1, \underline{f})|f(1, \bar{f})\nu e^+(\underline{f}, \bar{m})g(2)|g(1)\nu f(2, \underline{f})\mu^+(\bar{f}, \underline{m})|g(2)f(2, \bar{f})\nu\mu^-(\underline{f}, \underline{m})]$$

$$[e^-(\bar{f})\nu g(1, \underline{m})f(\underline{f})|f(\bar{f})e^+(\underline{f}, \bar{m})\nu g(2)|g(2)\nu\mu^+(\bar{f}, \underline{m})\mu^-(\underline{f}, \underline{m})]$$

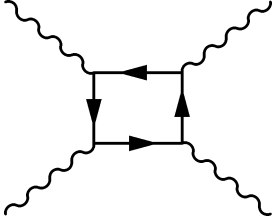
$$[e^-(\bar{f}, \bar{m})\nu g(2)f(\underline{f})|f(\bar{f})\nu g(1, \underline{m})e^+(\underline{f}, \bar{m})|g(2)\nu\mu^+(\bar{f}, \underline{m})\mu^-(\underline{f}, \underline{m})]$$

$$[e^-(\bar{f}, \bar{m})e^+(\underline{f}, \bar{m})\nu g(1)|g(1)\nu\mu^+(\bar{f}, \underline{m})f(\underline{f})|f(\bar{f})\nu g(2, \underline{m})\mu^-(\underline{f}, \underline{m})]$$

$$[e^-(\bar{f}, \bar{m})e^+(\underline{f}, \bar{m})\nu g(1)|g(1)\nu\mu^-(\underline{f}, \underline{m})f(\bar{f})|f(\underline{f})\nu g(2, \underline{m})\mu^+(\bar{f}, \underline{m})]$$

3 Simplifications

Some Feynman diagrams are very simple visually, but are very complex to describe using the strict notation outlined above. For many of these cases, simplifications of notation may be introduced. Take, for example, a diagram that contributes to photon-photon scattering:



In a strict use of notation, this diagram would be written as

```
[\gamma(1,\incoming)f(1,\arrowin)vf(2,\arrowout)|  
\gamma(2,\incoming)f(2,\arrowin)vf(3,\arrowout)|  
f(3,\arrowin)vg(3,\outgoing)f(4,\arrowout)|  
f(4,\arrowin)v\gamma(4,\outgoing)f(1,\arrowout)]
```

This complex description belies the true simplicity of the diagram. A more succinct description would treat the entire fermion loop like a vertex, $v(\text{A fermion loop})$. This reduces the number of effective vertices from four to one, and it eliminates the need for the clarifying commands `\incoming` and `\outgoing`. The result is

```
[\gamma(1)\gamma(2)v(\text{A fermion loop})\gamma(2)\gamma(3)]
```